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Research Description: In the present reporting period Tzavaras's work concentrated in the study of self-similar viscous and fluid-dynamic limits. The invariance of hyperbolic systems of conservation laws under dilations of coordinates is a key property underlying much of the current theory. Viscous perturbations introduce an additional parabolic scale and this invariance is lost. Understanding how the two scales interact for small viscosities is an important step in the process of studying viscous limits for general solutions. One illuminating step in that direction is to consider the Riemann problem and to study artificial regularizations, rigged so as to preserve the invariance under dilations of coordinates and the entropy structure of the system. It provides information on how viscosity regularizes the whole wave fan emerging from Riemann data. At the technical level, it leads to study of singular perturbations for non-autonomous boundary-value problems, and the limiting process involves variation estimates.

Our goal is to first develop technique and understanding of how the self-similar viscous limits work in various instances, when there is understanding of the expected behavior of the limiting equations. This will provide information in two directions: (i) in addressing problems that are not accessible by traditional techniques, like for instance non-conservative hyperbolic equations, and (ii) the information obtained via this approach will help understanding aspects of the difficult problem of viscous limits for systems of conservation laws. A technical description of completed projects, supported by the Office of Naval Research under contract N00014-93-0015, follows.

[a] *Wave structure induced by fluid dynamic limits in the Broadwell model.* to appear in Arch. Rational Mech. Analysis.

In this article we study self-similar fluid dynamic limits for the Broadwell model with Riemann, Maxwellian data. This is a natural context to consider the effect of relaxation mechanisms on shock capturing. The question is what admissibility restrictions are entailed on shocks by relaxation. It turns out that the limiting solutions, as the mean free path

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goes to zero, satisfy a pair of conservation laws and consist of two wave fans separated by a constant state. Each wave fan is associated with one of the characteristic fields and is either a rarefaction wave or a shock wave. The shocks satisfy the Lax shock conditions and have the internal structure of a Broadwell shock profile. (see the 93 progress report for a technical description of this project).

[b] *Elastic as limit of viscoelastic response, in a context of self-similar viscous limits*, to appear in J. Diff. Equations.

The solution of the Riemann problem for the equations of elasticity

$$\partial_t u - \partial_x v = 0$$
$$\partial_t v - \partial_x \sigma(u) = 0 \ . \tag{E}$$

is traditionally effected by providing admissibility criteria on shocks, and was first constructed for materials with nonconvex equations of state by Wendroff (1972). In this article we consider the system of equations

$$-\xi \begin{pmatrix} u \\ v \end{pmatrix}' - \begin{pmatrix} v \\ \sigma(u) \end{pmatrix}' = \varepsilon \begin{pmatrix} 0 \\ (k(u)\, v')' \end{pmatrix} , \tag{SVE}$$

$$u(\pm\infty) = u_\pm \quad , \quad v(\pm\infty) = v_\pm$$

where $(u_\pm, v_\pm)$ arbitrary, $\sigma(u)$ strictly increasing but nonconvex, and $k(u) = 1/u$ for longitudinal motions or $k(u) = 1$ for shearing motions. We show that (SVE) admits solutions for any $\varepsilon > 0$ and study the behavior of such solutions as $\varepsilon \to 0$. It turns out, that families of such solutions are of uniformly bounded variation and the limit (u, v) has the property that $(u(x/t), v(x/t))$ solves the Riemann problem for (E). The main question is then to study the structure of the emerging solution. This turns out to be composed of two wave fans, each consisting of rarefactions, shocks and contact discontinuities, separated by constant states. At shocks the self-similar viscous solution has the internal structure of traveling waves for the equation (E)., and a condition, first postulated by Wendroff as admissibility criterion, is automatically satisfied. The emerging structure is that dictated by the traditional solution of the Riemann problem and justifies the method as an alternative for situations where the former does not work.

[c] *Work in progress:*

Variation bounds and interaction estimates for self-similar viscous limits in general systems of conservation laws, manuscript in preparation.

For systems of more than two equations there is very little work concerning viscous limits and no comprehensive result. One problem appears to be lack of understanding of wave interactions, that are effected through diffusion now. To understand this issue we have been studying the system

$$-\xi U'_\epsilon + F(U_\epsilon)' = \varepsilon U''_\epsilon$$

$$U_\epsilon(\pm\infty) = U_\pm$$

The main step that been resolved is : Suppose that $F(U)$ corresponds to a strictly hyperbolic system and assume that we have a family of solutions $\{U_\epsilon\}$ that is of uniformly in ε small oscillation. Then the family is also of uniformly bounded variation. Performing this step requires understanding of wave interactions, and establishing a representation formula for U'_ϵ in terms of elementary waves. I am currently studying whether one can use this result to obtain an existence proof for the Riemann problem via this approach.

I have also initiated a collaboration with Ph. Lefloch (Ecole Polytechnique) to study systems of nonconservative hyperbolic equations. In a paper of DalMasso, Murat and Lefloch, there is proposed a definition for shocks in nonconservative equations. The question is whether we can understand this definition by using self-similar viscous limits, whose applicability and the techniques required do not depend on the equations been in conservative formulation. I expect that this issue will be of interest for several engineering problems where models that are not in conservative form are utilized.